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Silicon-based NMR quantum computer using single electron

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Abstract. A new NMR quantum computer scheme based on semiconductor nanostructure is proposed. Energy spectrum of the spin-qubit system was obtained. Methods of single- and two-qubit gates realization are discussed. This variant of quantum computer is more simple to control and more coherent.

One of the most promising ideas for the real quantum computer construction is creating artificial multispin systems in semiconductor nanostructures with the help of individual addressing to any qubit. For the last time a lot of theoretical suggestions, i.e. [1, 2] of such idea were discussed. Unfortunately, there are many experimental difficulties for modern nanotechnology to create even quantum computer prototype. We are speculating a new semiconductor quantum computer variant, which looks more realistic to construct.

In the considering approach it is suggested to use silicon-based structure, where donor atoms of stable phosphorus isotope ^{31}P are implanted into the spinfree silicon isotope ^{28}Si. Donor atoms replace silicon ones in knots of crystal lattice. Every donor atom with nuclear spin in semiconductor structure is supposed to be placed regularly with adequate accuracy, when metallic gates must control the number of electrons in such quantum dots and their interaction with the nucleus of the donor atom. Consider the cell of this quantum computer consisting of two qubits. With the help of changing of the electric potential on the metallic gates one can control the electron density near the nucleus. It is possible to leave only one valent electron in the considering cell, when other electrons must be displaced from this area. This electron will interact with its "own" nucleus, as well as with the other. That is why the Hamiltonian of the system is:

$$\hat{H} = -2\mu_B \vec{B}\hat{\vec{S}} - g_N \mu_N \vec{B}\left(\hat{\vec{I}}_a + \hat{\vec{I}}_b\right) + A_a\left(\hat{\vec{I}}_a\hat{\vec{S}}\right) + A_b\left(\hat{\vec{I}}_b\hat{\vec{S}}\right) \tag{1}$$

when $\mu_B = 9.27 \cdot 10^{-24}\ J/T$ — Bohr magneton, $\mu_N = 5.05 \cdot 10^{-27}\ J/T$ — nuclear magneton, $g_N = 2.26$ — Lande's factor for ^{31}P, $\vec{B}$ — the magnetic field, $\hat{\vec{S}}$ — electron spin operator, $\hat{\vec{I}}_a$ and $\hat{\vec{I}}_b$ — spin operators for nuclei a and b, A_a and A_b — hyperfine interaction constants, depending on gate potentials. We choose eigen states of the Hamiltonian (1) in the strong magnetic field to be the basis states. Designating spin projections of the electron and the nuclei by arrows up ($\uparrow$) and down ($\downarrow$), we have (the first arrow concerns the electron, the second — nucleus a, the third - nucleus b):

$$\begin{array}{ll} |1\rangle = |\uparrow\uparrow\uparrow\rangle & |5\rangle = |\downarrow\uparrow\uparrow\rangle \\ |2\rangle = |\uparrow\uparrow\downarrow\rangle & |6\rangle = |\downarrow\uparrow\downarrow\rangle \\ |3\rangle = |\uparrow\downarrow\uparrow\rangle & |7\rangle = |\downarrow\downarrow\uparrow\rangle \\ |4\rangle = |\uparrow\downarrow\downarrow\rangle & |8\rangle = |\downarrow\downarrow\downarrow\rangle \end{array} \tag{2}$$

Table 1. CNOT gate scheme realization.

Operation	The state of the qubit A	The state of the qubit B	The electron state
—	Ψ_A	Ψ_B	$\|0\rangle$
SWAP operation between A qubit and the electron	$\|0\rangle$	Ψ_B	Ψ_A
Adiabatic transfer of the electron from nucleus A to nucleus B	—	—	—
CNOT operation between the electron and qubit B	$\|0\rangle$	Ψ_B	$\mathrm{CNOT}\|\Psi_A, \Psi_B\rangle$
Adiabatic transfer of the electron from nucleus B to nucleus A	—	—	—
SWAP operation between the electron and A qubit	$\mathrm{CNOT}\|\Psi_A, \Psi_B\rangle$	Ψ_B	$\|0\rangle$

Symmetric matrix 8×8, corresponding to the matrix of eigen states of Hamiltonian (1), in the considered basis, is

$$\begin{pmatrix}
\mu_B B - g_N \mu_N B + \frac{A_a + A_b}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\mu_B B + \frac{A_a - A_b}{4} & 0 & 0 & A_b/2 & 0 & 0 & 0 \\
0 & 0 & -\mu_B B + \frac{-A_a + A_b}{4} & 0 & A_a/2 & 0 & 0 & 0 \\
0 & 0 & 0 & -\mu_B B + g_N \mu_N B - \frac{A_a + A_b}{4} & 0 & A_a/2 & A_b/2 & 0 \\
0 & A_b/2 & A_a/2 & 0 & \mu_B B - g_N \mu_N B + \frac{A_a + A_b}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & A_a/2 & 0 & \mu_B B + \frac{A_a - A_b}{4} & 0 & 0 \\
0 & 0 & 0 & A_b/2 & 0 & 0 & \mu_B B + \frac{-A_a + A_b}{4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_B B + g_N \mu_N B - \frac{A_a + A_b}{4}
\end{pmatrix}.$$

It is worth of noting that using only one electron to be the connection between two nuclei significantly decreases decoherence processes in quantum computer due to electron-

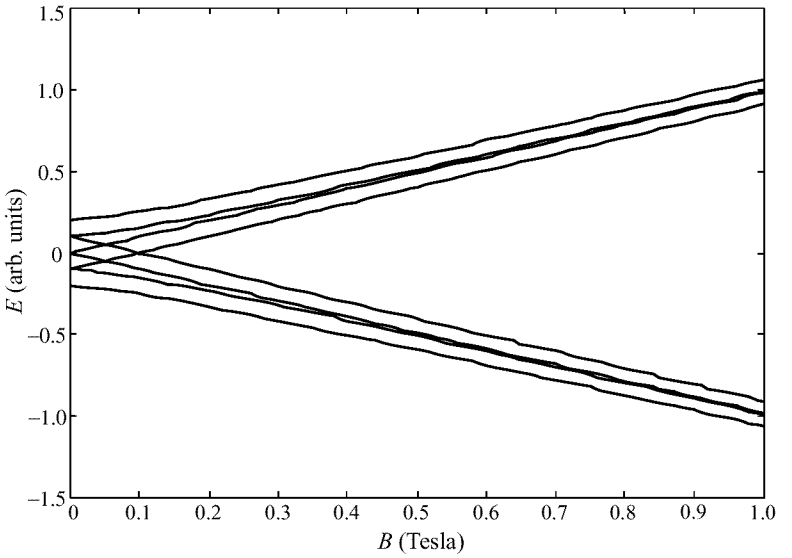

Fig. 1. Energy spectrum of the system, described by the Hamiltonian (1).

phonon interaction. Also, owing to other valent electrons absence metallic gates do not influence them (including other donors' electrons), that is why considered scheme is more simple to control. Quantum single-qubit operations (for example, NOT) are made with the help of radio-frequency pulses with resonant frequency. As concerns two-qubit operations (for example, CNOT), their realization is shown in Table 1.

One of the most important problems in quantum computing is the problem of the final state measurement. Unfortunately, the absence of the energy level anticrossing effect (Fig. 1) prevent using it to measure the final state, as it was made in [3, 4]. We suppose recent suggestion [5] to be the promising to solve this problem. A "turnstyle" can detect a current of electrons with definite polarization. This scheme can be used not only for single spin state measurements, but for spin system state measurement as well. When one wants to measure any qubit's state, SWAP operation between that nucleus spin-qubit and the electron must be made. After that electron spin state measurement (measurement of the spin polarization) completes the measurement process. Finally, we propose a novel qubit based on NMR of donor nuclei in semiconductor nanostructures. It is more coherent and more simple to control compared to proposal of Kane [1].

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